**La relation d’Euler - sous-titres**

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00:00:26,000 --> 00:00:30,999

Hello everyone. Bienvenue à

this topology session,

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00:00:31,000 --> 00:00:36,500

et plus spécifiquement aux graphes planaires,

which are important in combinatorics.

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00:00:37,000 --> 00:00:39,700

Donc commençons cette section.

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Today, you will need

small pieces of paper;

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00:00:45,000 --> 00:00:50,000

some pencils (not necessarily with colour)

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00:00:51,000 --> 00:00:54,000

a ruler; and squared

paper.

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00:00:54,500 --> 00:00:58,500

Like I've already said, we will study

planar graphs.

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The best way to start is

to draw some.

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So, let me explain, how

to draw them.

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To draw a planar graph, it is enough

to first draw the vertices.

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which I will do with green circles.

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Then, you have to connect the vertices

with edges.

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It is allowed to connect a vertex

with itself.

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we can just draw a loop like this one.

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we can also draw multiple edges

between two vertices, like this.

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This is allowed!

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To draw a planar graph, you have to

follow two rules.

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The first rule is

that the edges you draw

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may not intersect one another.

So, this situation is not allowed.

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Rule no. 2: In the end, the graph

we get

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(we get a graph the moment we have

drawn the edges and vertices)

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00:02:08,000 --> 00:02:10,900

the graph must be connected.

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00:02:11,000 --> 00:02:14,500

That means, that if we want to Walt from

one vertex to another

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00:02:14,600 --> 00:02:19,900

there must exist a path

between them.

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Apart from there in this example, there are two

groups of vertices

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without a connection between them,

so the graph is disconnected.

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To make this graph feasible, it is enough

to draw one edge between these two groups.

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I recommend you, to draw it

between 5 and 10.

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So, please pause this video

take your small pieces of paper

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and draw a graph

on each of them.

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There it is! For example, I have drawn eight

graphs on eight different paper sheets.

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Take this graph for example.

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There are different things to

see at this graph.

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00:03:03,000 --> 00:03:05,400

First, we can count the

vertices

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which are the green circles here.

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00:03:08,300 --> 00:03:12,000

Let S be the number of vertices.

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To count the vertices

of a more complicated graph

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I recommend you to

number the vertices

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while counting them.

For example: Here we have 1, 2, 3, 4, 5, 6 vertices.

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00:03:30,600 --> 00:03:33,500

Thus, S equals 6.

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Denote A the number of edges.

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Like for the vertices,

if the graph is more complicated,

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you can mess up counting (by forgetting an edge

or by counting one twice)

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so I recommend you to count

the edges

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00:03:48,800 --> 00:03:51,000

by crossing out each edge you counted.

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00:03:51,100 --> 00:04:05,000

For example, here we have 1, 2, 3, 4, 5, 6, 7, 8,

9, 10. edges. Hence, A = 10.

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Finally, the last objects, we have in

a planar graph, are the faces.

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To count the faces,

I recommend to start at the middle of an edge

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00:04:14,600 --> 00:04:18,500

for example this one,

you follow it in one direction

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00:04:18,600 --> 00:04:22,000

for example this one.

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00:04:22,500 --> 00:04:26,500

And finally come back to where we started.

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This circle

defines a face of your graph.

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00:04:32,000 --> 00:04:34,500

Let F be the number of faces.

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00:04:46,000 --> 00:04:47,900

Do not forget the outer face:

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00:04:48,000 --> 00:04:51,500

If you start at this point for example

and you follow the graph

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00:04:51,550 --> 00:04:56,500

in one direction, for example this one,

you can see, you will have to

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00:04:56,550 --> 00:05:02,000

follow the outer line of the graph.

That defines the outer face.

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Do not forget to count this face equally.

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00:05:08,000 --> 00:05:14,000

So, how many faces are there?

There are 1, 2, 3, 4, 5 and 6. So, F=6.

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I want to show you this example graph

because it is a little bit special.

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Here, we have edges,

that are a bit isolated.

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Je vais vous montrer un peu pour que vous ne

vous trompiez pas.

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Là vous voyez que l'on passe encore une

deuxième fois le long de cette arête

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et vous voyez qu'à la fin, on va revenir au

point de départ.

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Et donc là, la face à l'extérieur a une forme

un petit peu bizarre,

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00:05:50,000 --> 00:05:51,500

mais c'est tout de même une face.

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00:05:51,600 --> 00:05:57,000

Ce que je vous propose, c'est de prendre tous

vos graphes et de compter pour chaque graphe

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00:05:57,100 --> 00:06:01,500

le nombre de sommets, le nombre d'arêtes et le

nombre de faces, et de l'écrire en-dessous.

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00:07:15,000 --> 00:07:19,200

Maintenant que vous avez compté le nombre

de sommets, d'arêtes et de faces,

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00:07:19,300 --> 00:07:21,500

d'abord vous allez tracer deux axes :

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00:07:21,600 --> 00:07:27,500

un axe vertical avec votre règle graduée

et un axe horizontal.

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00:07:28,000 --> 00:07:33,900

L'axe horizontal, vous allez le graduer

entre 1 et une vingtaine,

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et de même pour l'axe vertical.

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00:07:35,800 --> 00:07:39,200

Les graduations doivent être régulièrement espacées.

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00:07:39,400 --> 00:07:41,800

Pour chaque feuille,

vous allez prendre votre graphe,

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00:07:41,900 --> 00:07:44,900

et l'axe horizontal correspond au nombre d'arêtes.

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00:07:45,000 --> 00:07:48,900

L'axe vertical correspond au nombre de faces plus le nombre de sommets.

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00:07:49,000 --> 00:07:54,900

A vaut 10, donc on va se placer sur l'axe horizontal au niveau du chiffre 10.

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Si vous calculez S+F, ça fait 6+6 donc 12, donc vous allez monter jusqu'à la graduation 12.

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00:08:02,600 --> 00:08:05,500

Et vous allez tracer une croix ici.

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00:08:05,600 --> 00:08:06,500

Mettez en pause la vidéo,

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00:08:06,600 --> 00:08:10,000

et pour chaque graphe que vous avez dessiné, vous allez tracer un point.

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00:08:56,000 --> 00:08:59,000

There it is!

Vous avez obtenu des points sur un gaphe.

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00:08:59,100 --> 00:09:03,500

Maintenant, je vais vous demander de mettre en pause la vidéo (encore une fois)

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00:09:03,600 --> 00:09:08,900

et de discuter entre vous de la particularité de ces points que vous obtenez.

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00:09:09,000 --> 00:09:12,000

Savez-vous pourquoi il y a une telle caractéristique ?

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Alors vous avez probablement remarqué

que tous les points, normalement, sont alignés.

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C'est-à-dire que, si vous prenez votre règle,

et que vous tracez une droite,

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00:09:29,600 --> 00:09:33,500

vous allez arriver à tracer une droite qui

passe par tous les points.

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00:09:33,600 --> 00:09:37,000

Je vous invite à tracer cette droite.

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00:09:43,000 --> 00:09:50,000

Et vous allez voir que cette droite passe,

au niveau de l'axe vertical,

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00:09:50,100 --> 00:09:53,100

au niveau du chiffre 2 (normalement).

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00:09:58,000 --> 00:10:01,000

Comme équation, l'équation suivante,

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00:10:01,100 --> 00:10:18,000

c'est-à-dire F+S = A+2

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00:10:20,000 --> 00:10:24,500

Voilà.

Ou alors, écrit d'une autre manière,

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00:10:24,600 --> 00:10:29,000

donc de manière équivalente, on peut aussi

faire passer le A de l'autre côté,

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00:10:30,000 --> 00:10:39,000

et donc on obtient F-A+S=2

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Pourquoi a-t-on une telle équation ?

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00:10:45,000 --> 00:10:50,000

Je vous laisse en discuter quelques instants

entre vous, donc mettez en pause la vidéo.

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Now, that you have discussed,

if the formula is true

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The formula F - A + S = 2

I am going to prove it.

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00:11:14,500 --> 00:11:18,600

That formula is called

the Euler characteristic.

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d'après le nom du mathématicien Leonhard Euler.

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00:11:22,200 --> 00:11:25,000

Voilà, merci d'avoir suivi cette vidéo !

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00:11:25,800 --> 00:11:31,000

Sachez que des formules similaires existent

aussi pour des graphes non-planaires

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00:11:31,200 --> 00:11:35,100

(ce sont des graphes où l'on peut autoriser

aussi des croisements)

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00:11:36,000 --> 00:11:39,900

et cette relation d'Euler est vraiment

universelle et c'est pour ça que

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00:11:40,000 --> 00:11:42,000

je la trouve très belle.

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00:11:43,000 --> 00:11:46,900

Les chercheurs et les chercheuses qui font de

la combinatoire l'utilisent très souvent

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00:11:47,000 --> 00:11:51,500

pour classer les graphes qu'iels étudient.

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Merci beaucoup d'avoir suivi cette vidéo et

à bientôt !